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String field theory at large B-field and non-commutative geometry

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Abstract: In the search for the exact minimum of the tachyon potential in the Witten's cubic string field theory we try to learn as much as possible from the string field theory in the large B-field background. We offer a simple alternative proof of the Witten's factorization, carry out the analysis of string field equations also for the non-commutative torus and find some novel relations to the algebraic K-theory. We note an intriguing relation between Chern-Simons and Chern classes of two non-commutative bundles. Finally we observe a certain pattern which enables us to make a plausible conjecture about the exact form of the minimum.

Keywords: Bosonic Strings, D-branes, Non-Commutative Geometry.

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Contents

1. Introduction

One of the mysteries of string theory which has not been so far fully understood mathematically is the Sen's conjecture [1]. According to it there are solitonic solutions to string field theory equations of motion for which the action attains rather special values which coincide with the mass differences between various branes known from different considerations. The conjecture was tested with amazing precision [2, 3] using the numerical method developed in [4]. Only very recently it has been proved analytically in the framework of the so called background independent string field theory [35, 36, 37].

In spite of this recent progress the fact that the Witten-Chern-Simons string field theory [5] has this property still remains rather miraculous and lacks understanding. One is tempted to believe that it is the non-commutative topological nature of the action which may help to prove the Sen's conjecture in the realm of this theory. Life is however not too easy. The main formal obstacle to use the methods of non-commutative geometry [6] appears to be the lack of any satisfactory definition of the string field algebra which is both closed and associative. One may also question whether it is the right strategy to take since an analogous conjecture is believed to be true also for the superstring [7]–[10] where no non-commutative geometric formulation is known.

Regarding the study of lower-dimensional branes there were two basic approaches in the literature. In the first one [11, 12, 13] they were studied numerically as solitons in the string field theory. A systematic numerical method — modified level expansion scheme — was developed in [13]. The second approach [14, 15, 16] was based on the observation that in the large B-field limit the effective action for the tachyon field admits simple solutions which can be interpreted as lower-dimensional branes whose tensions exactly reproduce the known results. This last series of results was beautifully put to the string field theory setting by Witten [17], who showed the factorization of the algebra in the large B-field limit. The beauty of his work lies mainly in the fact that two hitherto unrelated occurrences of non-commutative geometry in string theory were shown to be intimately connected.

We feel therefore that a good starting point to understand the mysteries of the string field algebra and action is to start by examining it carefully in the limit of large B-field. In the course of this work we will try to pay special attention to the various non-commutative geometric aspects. It should not be perhaps surprising that algebraic K-theory will show up to play some role since it is one of the key building blocks of non-commutative geometry in its abstract setting. The role we find here is however quite different from the one recently discussed in a similar context in [24, 25]. They study the K-theory along the worldvolume of the brane whereas we focus on the transverse directions. The K-theory we discuss is the one which previously appeared in [26]. For a friendly introduction to K-theory see [27] and for the overview of applications to string theory see [28, 29].

The paper is organized as follows. Section 2 is devoted to the rederivation of the Witten's factorization in the operator language which is much simpler and more transparent. The basic ingredient here is the three string vertex in the B-field background constructed in [20, 21]. In section 3 we observe natural correspondence between D-brane decays and algebraic K-theory. We deal with two particular cases. In the first one the B-field is put on the plane $\mathbb{R}^2$. Here we can utilize the GMS [14] construction of the projectors. In the second case we consider the B-field on a two torus $\mathbb{T}^2$ where it becomes necessary to use the Powers-Rieffel projectors [32] or some variant thereof. We find quite an intriguing connection between invariants of the non-commutative bundle over the torus defined by the choice of a projector and non-commutative bundle defined by the choice of a string field connection. Section 4 contains observation about mutual relations between various solutions to string field equations. It formally looks as a gauge transformation, but the relevant isometry is nonunitary. This leads at the end to a plausible proposal for the exact minimum. Concrete examples and numerical tests are postponed to the future work.

2. String field algebra at non-zero B-field

All the degrees of freedom of string field theory are contained in the string field

$$\Psi = \int d^{26}p \, (t(p)c_1 + A_\mu(p)\alpha^\mu_{-1}c_1 + \cdots)|0,p\rangle \qquad (2.1)$$

which is an element of the Fock space of the first quantized string theory. It is governed by the Chern-Simons type of an action

$$S[\Psi] = -\frac{1}{\alpha' G_o^2}\left(\frac{1}{2}\langle\, \Psi, Q\Psi\,\rangle + \frac{1}{3}\langle\, \Psi, \Psi * \Psi\,\rangle\right). \qquad (2.2)$$

The non-commutative star multiplication originally defined in terms of gluing of strings [5] was formulated in the operator language in [18, 19] and the B-field was taken into account in [20, 21]. It is defined through

$$\Psi_1 * \Psi_2 = bpz\left(\langle V|\Psi_1 \otimes \Psi_2\right), \tag{2.3}$$

where bpz denotes the bpz conjugation in conformal field theory and

$$\begin{aligned}\langle V| = &\left(\frac{3\sqrt{3}}{4}\right)^3 \delta(p^{(1)}+p^{(2)}+p^{(3)})\langle\tilde{0}|\otimes\langle\tilde{0}|\otimes\langle\tilde{0}|\times \\ &\times\exp\left(\sum_{m,n=0}^{\infty}\frac{1}{2}\alpha_n^{(r)\mu}N_{nm}^{rs}\alpha_m^{(s)\nu}G_{\mu\nu}+\sum_{m=0,n=1}^{\infty}c_n^{(r)}X_{nm}^{rs}b_m^{(s)}-\frac{i}{2}\theta_{\mu\nu}p^{(1)\mu}p^{(2)\nu}\right).\end{aligned} \tag{2.4}$$

The Neumann coefficients N_{nm}^{rs}, X_{nm}^{rs} are reviewed in [22]. The vacuum $\langle\tilde{0}|$ is related to the standard $\mathrm{Sl}(2,\mathbb{R})$ invariant vacuum $\langle 0|$ through $\langle\tilde{0}| = \langle 0|c_{-1}^{(i)}c_0^{(i)}$ where $i=1,2,3$ label one of the three Fock spaces in the tensor product. As usual $\alpha_0^\mu = \sqrt{2\alpha'}p^\mu$.

The effective open string coupling constant, open string metric and the noncommutativity parameter [23] are given by

$$\begin{aligned}G_o &= g_o\left(\frac{\det G}{\det(g+2\pi\alpha' B)}\right)^{1/4},\\ G_{\mu\nu} &= g_{\mu\nu}-(2\pi\alpha')^2(Bg^{-1}B)_{\mu\nu},\\ \theta^{\mu\nu} &= -(2\pi\alpha')^2\left(\frac{1}{g+2\pi\alpha' B}B\frac{1}{g-2\pi\alpha' B}\right)^{\mu\nu}.\end{aligned} \tag{2.5}$$

These effective parameters also appear in the formula for Virasoro generators (and therefore in the BRST charge Q) and in the commutation relations for the Fock space generators

$$\begin{aligned}[\alpha_m^\mu, \alpha_n^\nu] &= m\delta_{m+n,0}G^{\mu\nu},\\ [x^\mu, x^\nu] &= \theta^{\mu\nu},\\ [p^\mu, x^\nu] &= -iG^{\mu\nu}.\end{aligned} \tag{2.6}$$

Large B-field limit. Now take the limit $B\to\infty$ keeping fixed all closed string parameters (including the open string coupling constant g_o but not the effective one G_o). To make things more transparent set $B = tB_0$ and take $t\to\infty$ as in [17]. The effective parameters clearly depend on t as

$$\begin{aligned}G_o &\sim G_{o\,0}t^{r/2},\\ G^{\mu\nu} &\sim G_0^{\mu\nu}t^{-2},\\ \theta^{\mu\nu} &\sim \theta_0^{\mu\nu}t^{-1},\end{aligned} \tag{2.7}$$

where r denotes the rank of the B-field and for the brevity let us assume that it is maximal. Altogether the t dependence enters at two places: first in the commutation relations (2.6) for the Hilbert space operators and then also explicitly in the definition (2.4) of the star product.

To see the change in the structure of the string field algebra we have to rescale all Fock space operators in such a way that their commutation relations do not depend on t. (We are then sure that we are studying different star products on the same space.) The rescaling which does that is

$$\begin{aligned} \alpha^\mu_m &\longrightarrow \tilde{\alpha}^\mu_m = t\,\alpha^\mu_m\,, \qquad (m \neq 0)\,,\\ p^\mu &\longrightarrow \tilde{p}^\mu = t^{3/2} p^\mu\,,\\ x^\mu &\longrightarrow \tilde{x}^\mu = t^{1/2} x^\mu\,. \end{aligned} \tag{2.8}$$

After this rescaling the exponent in the vertex (2.4) takes the simple form

$$\begin{aligned} &\sum_{m,n=1}^{\infty} \frac{1}{2}\tilde{\alpha}^{(r)\mu}_n N^{rs}_{nm} \tilde{\alpha}^{(s)\nu}_m G_{0\mu\nu} + \frac{1}{\sqrt{t}} \sum_{n=1}^{\infty} \sqrt{\frac{\alpha'}{2}}\tilde{\alpha}^{(r)\mu}_n (N^{rs}_{n0} + N^{rs}_{0n})\tilde{p}^{(s)\nu} G_{0\mu\nu} + \\ &\quad + \frac{1}{t}\alpha' \tilde{p}^{(r)\mu} N^{rs}_{00} \tilde{p}^{(s)\nu} G_{0\mu\nu} + \sum_{m=0,n=1}^{\infty} c^{(r)}_n X^{rs}_{nm} b^{(s)}_m - \frac{i}{2}\theta_{0\mu\nu}\tilde{p}^{(1)\mu}\tilde{p}^{(2)\nu}\,. \end{aligned} \tag{2.9}$$

We see that in the large t limit the terms which couple α oscillators with momenta p vanish but the whole star product nevertheless remains non trivial. Now the generic string field is the sum of terms

$$a\, e^{ik^\mu x^\nu G_{\mu\nu}}|0\rangle = a\, e^{i\tilde{k}^\mu \tilde{x}^\nu G_{0\mu\nu}}|0\rangle\,, \tag{2.10}$$

where $a \in \mathcal{A}_0$ is in the zero momentum subalgebra. It is then obvious that the star product respects the tensor product structure.

$$a_1 e^{i\tilde{k}_1^\mu \tilde{x}^\nu G_{0\mu\nu}}|0\rangle * a_2 e^{i\tilde{k}_2^\mu \tilde{x}^\nu G_{0\mu\nu}}|0\rangle = (a_1 * a_2) e^{-\frac{i}{2}\tilde{k}_1^\mu \tilde{k}_2^\nu \theta_{0\mu\nu}} e^{i(\tilde{k}_1+\tilde{k}_2)^\mu \tilde{x}^\nu G_{0\mu\nu}}|0\rangle\,. \tag{2.11}$$

Recalling the structure of the BRST operator it is also obvious that after this rescaling in the limit $t \to \infty$ all the terms with momentum operators vanish and therefore it acts only on the $\mathcal{A}_0$ component. In conclusion the full string field algebra looks as

$$\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1\,, \tag{2.12}$$

where $\mathcal{A}_0$ is the complicated stringy subalgebra of the string states of zero momentum in the non-commutative directions and non-zero momentum in the commutative ones. The second factor $\mathcal{A}_1$ is the algebra generated by the functions e^{ikx} using the Moyal product. Its precise content, K-theory and physical applications in the important cases of (compactified) Moyal plane and non-commutative torus will be our primary concern in the next section.

3. Solutions of the string field equations and K-theory

As the first case let us consider the flat Minkowski space with $g_{\mu\nu} = \eta_{\mu\nu}$ and for simplicity assume that the rank of the B-field is two. This was studied in [14, 15, 16]. For the algebra $\mathcal{A}_1$ of functions on the non-commutative plane let us take the Schwarz space $\mathcal{S}(\mathbb{R}^2)$. The associated algebra of Weyl ordered operators generates the algebra of the trace-class operators whose norm closure is the algebra $\mathcal{K}(\mathcal{H})$ of compact operators on a separable Hilbert space $\mathcal{H}$ [31]. This algebra does not contain the identity, we may wish to add it by hand. This formally corresponds to the one point compactification of the Moyal plane. Thus we have up to an isomorphism

$$\mathcal{A}_1 = \mathcal{K} \oplus \mathbb{C}\mathcal{I}\,. \tag{3.1}$$

The K_0 group of this algebra which will play some role later is

$$K_0(\mathcal{A}_1) = \mathbb{Z} \oplus \mathbb{Z}\,. \tag{3.2}$$

For a general algebra it is defined as the additive group of formal differences of certain equivalence classes of projectors. For a detailed exposition see [27].

Let us discuss now some solutions to the string field equation of motion in the background of large B-field. From the action (2.2) it takes the form

$$Q\Psi + \Psi * \Psi = 0\,. \tag{3.3}$$

The basic solution is $\Psi = A_0 \otimes \mathcal{I}$. It is the famous solution describing the decay of the D25-brane which was first investigated numerically in [2]. The value of the string field action per unit time[1] is (using the Sen's conjecture for $B = 0$)

$$S[A_0 \otimes \mathcal{I}] = 2\pi\alpha' BM \tag{3.4}$$

in accord with the Sen's conjecture for B large. Here M stands for the D25-brane mass in the absence of any B-field

$$M = \frac{1}{2\pi^2}\frac{1}{\alpha' g_o^2}\int \sqrt{g}\, d^{25}x\,. \tag{3.5}$$

The factor $2\pi\alpha' B$ comes from the effective open string coupling constant and from the normalization of the inner product

$$\langle 0,0|c_{-1}c_0c_1|0,0\rangle = \int \sqrt{G}\, d^{26}x \tag{3.6}$$

and accounts precisely for the change in the mass of the D25-brane due to the background B-field. Note that there are some subtleties since the mass M diverges. To

[1] Throughout the whole paper we are interested in time independent configurations and hence the word action will always mean the action per unit time.

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make it finite, we should introduce some cutoff, which however spoils the structure of the algebra. Nevertheless the simplicity of the GMS construction partially justifies this slightly heuristic treatment. The more careful treatment of the non-commutative torus will be given later.

As was noticed by [16] on the level of the low energy action and by Witten [17] from the string field theory point of view one can get whole family of new solutions of the form $A_0 \otimes \rho$ where $\rho \in \mathcal{A}_1$ is any projector. Suppose now for a while that ρ is a projector onto a finite n-dimensional subspace of $\mathcal{H}$. For all of these solutions one can easily calculate the value of the string field action using the Sen's conjecture for the D25-brane without any B-field. Let us list some of them in the suggestive form

Solution	Value of the action	Interpretation
$A_0 \otimes \mathcal{I}$	$2\pi\alpha' BM$	$D25 \longrightarrow vac$
$A_0 \otimes (\mathcal{I} - \rho)$	$\left(2\pi\alpha' B - n\frac{\alpha'}{R^2}\right) M$	$D25 \longrightarrow nD23$
$A_0 \otimes \rho$	$n\frac{\alpha'}{R^2} M$	$nD23 \longrightarrow vac$

In order to get finite results we had to regularize the area of the Moyal plane (in closed string metric) to be $(2\pi R)^2$. We also need the formula $\int d^2x\, \rho(x) = 2\pi\theta n$ from [14]. The values of the action for the above solutions exactly correspond to the decay energies between various D-brane systems. This leads to the interpretation listed in the last column. One can readily enlarge the above table to include various lower-dimensional branes and also to introduce Chan-Paton factors introducing thus more D25-branes.

Sum of any two solutions (or two projectors) is a solution (or a projector) only if the two projectors are orthogonal. To be able to add any two projectors K-theory does what in physical terms is called introducing Chan-Paton factors. At this point it should be clear that we can also interpret the K_0 elements as formal differences of branes. The string field action clearly acts as a homomorphism on this group. This interpretation of K_0 is somehow reminiscent of the situation in IIB theory [30] where the elements are formal differences of *bundles* on branes and antibranes. Nonetheless, the fact that it is a completely different group is more than obvious. It may seem that the K-theory we are talking about is just an artefact of the "hand-made" algebra (3.1). By looking at the example of the (more realistic) non-commutative torus we shall try to convince the reader that there is something deeper going on.

Non-commutative torus. Let us briefly discuss the case of the non-commutative torus. Here the relevant algebra $\mathcal{A}_1$ is the well-known rotational algebra A_θ. Its K_0 group is the same as for the compactified Moyal plane above. Unfortunately in this case the beautiful construction [14] of all the projectors breaks down even though one still has a homomorphism from the algebra $\mathcal{A}_1$ to the space of bounded operators by an analog of the Weyl quantization formula. Some representatives of all the equivalence classes of projectors were nevertheless constructed by Rieffel [32]. The

Powers-Rieffel projector on the torus $[0,2\pi]^2$ takes the form (in the representation by ordinary functions)

$$p(x_1,x_2) = 2\cos(x_1) g\left(e^{i(x_2+\theta/2)}\right) + f\left(e^{ix_2}\right), \tag{3.7}$$

where f and g are two functions satisfying certain relations. These can be chosen to be sufficiently smooth if one wishes. The trace on the non-commutative algebra $\mathcal{A}_1$ in the representation by ordinary functions with the Moyal product is just an ordinary integral over the torus (normalized by the total area) which gives precisely $\frac{\theta}{2\pi}$. From our point of view the only problematic feature of these solutions is that they are not well localized in one direction (in this case x_1). This prevents us from looking at those solutions as codimension two lump representing lower-dimensional brane. Nevertheless from our experience with the Moyal plane, we believe that there should exist also well-localized solutions with straightforward physical interpretation.

General theorem due to Pimsner and Voiculescu when combined with the Rieffel's construction [27, 32] states that the range of the trace on projections in $\mathcal{A}_1$ is exactly $(\mathbb{Z} + \frac{\theta}{2\pi}\mathbb{Z}) \cap [0,1]$. The unusual normalization factor $\frac{1}{2\pi}$ comes from requiring the standard form (2.11) of the star product. Calculating the string field action for the solution $A_0 \otimes p$ one gets

$$S[A_0 \otimes p] = 2\pi\alpha' BM \operatorname{Tr} p = 2\pi\alpha' BM \left(m - \frac{\theta}{2\pi}n\right), \tag{3.8}$$

where $m,n \in \mathbb{Z}$ are such that

$$m - \frac{\theta}{2\pi}n \in [0,1]\,. \tag{3.9}$$

We see that for $m=1$ and $n \in \mathbb{N}$ not too large (such that the projector exists) we get *precisely* the same values as those for the Moyal plane above. It is perhaps curious to note that the theorem also asserts that even without introducing the Chan-Paton factors one can describe the decay of $m>1$ D25 branes into an appropriate number of D23-branes. This is not true for the Moyal plane case.

As we said above one may have doubts about the role of K-theory on the Moyal plane. But here on the non-commutative torus in order to find a single example of a projector we had to use the K-theoretical sources. Strikingly these projectors lead to the correct masses of D-branes, exactly as the GMS projectors. The projectors in non-commutative geometry are primarily used to define projective modules — a non-commutative generalization of vector bundles — which are naturally classified by K-theory. To end up this section we would like to make the following interesting remark. The string field action is the (secondary) Chern-Simons class of the non-commutative bundle defined by the connection which is the string field. In the large B-field limit when the algebra factorizes the action becomes equal up to a factor to the Chern class of a completely different non-commutative bundle over the torus specified by the choice of the projector p. We believe that further investigations may reveal beautiful interplay between these objects in non-commutative geometry.

4. Proposal for the exact solution of the tachyon potential

The proposal is based on the following simple observation: all the decays of D25-brane that are described in the large B-field limit by taking a non-zero projector $\rho \in \mathcal{A}_1$ are related to each other by a non-unitary isometry.[2] Of course it does not mean that they are in the same K-theory class since this isometry does not belong to $\mathcal{A}_1$. To give an example consider the solutions describing the decays D25 $\to$ *vac* and D25 $\to$ (n)D23 with projectors $\mathcal{I}$ and $\mathcal{I}-\rho$, respectively.

The isometry U which relates them as follows:

$$\begin{aligned} \mathcal{I}-\rho &= UU^\dagger\,, \\ \mathcal{I} &= U^\dagger U \end{aligned} \tag{4.1}$$

can be found in some cases explicitly. If for instance we take $\rho = |0\rangle + |1\rangle + \cdots + |n-1\rangle$ then U and $U^\dagger$ are the ordinary shift operators

$$\begin{aligned} U &= \sum_{m=0}^{\infty} |m+n\rangle\langle m|\,, \\ U^\dagger &= \sum_{m=0}^{\infty} |m\rangle\langle m+n|\,. \end{aligned} \tag{4.2}$$

The operator U is clearly non compact (and it is neither unity) so it does not belong to $\mathcal{A}_1$. Thus the projectors $\mathcal{I}$ and $\mathcal{I}-\rho$ do not have to belong to the same K-theory class. They would, however, if we were working with the algebra of all bounded operators.

Note that string field solutions representing the above decays are related by a formula which formally looks like a string field gauge transformation

$$A_0 \otimes (\mathcal{I}-\rho) = U(Q + A_0 \otimes \mathcal{I})U^\dagger\,. \tag{4.3}$$

The first term on the right hand side gives of course zero contribution since Q does not act on $\mathcal{A}_1$. More useful relation is obtained by multiplying with $U^\dagger$ and U on the left and right, respectively

$$A_0 \otimes \mathcal{I} = U^\dagger(Q + A_0 \otimes (\mathcal{I}-\rho))U\,. \tag{4.4}$$

Our conjecture is as follows: since the decays D25 $\to$ *vac*, D25 $\to$ nD23 and so on are related by gauge-like isometry transformation, it is natural to expect that in the full string theory also the trivial process D25 $\to$ D25 described by the zero string field is related to the others in a similar way. Thus we expect

$$A_0 = V^\dagger * QV \tag{4.5}$$

[2] An operator U for which $U^\dagger U$ is projector is called a partial isometry. Then automatically $UU^\dagger$ is a projector. If $U^\dagger U = \mathcal{I}$ then U is called an isometry.

for some V acting on $\mathcal{A}_0$ and satisfying

$$\begin{aligned} V^\dagger * V &= \mathcal{I}\,, \\ Q(V * V^\dagger) &= 0\,, \end{aligned} \tag{4.6}$$

where the star is now the stringy product (not the Moyal one) and the dagger means the usual star involution of the string field algebra. The last equation (which is of course also satisfied by U) was added in order to fulfill the equation of motion. Note that both conditions (4.6) could be replaced simply by $V * V^\dagger = \mathcal{I}$ but this is not favored by our analogy. It is straightforward to check the string field equations of motion (3.3) provided one can use the associativity of the algebra. This is however not a priori clear since V (in analogy with U) does not appear to be an element of the algebra. Indeed it is well known that when one tries to add some elements to the so far not properly defined string algebra one runs into problems with associativity anomalies [33, 34]. We end up by emphasizing the importance of clarification of these anomalies and by expressing the hope that it will be soon possible to confirm the above conjecture at least numerically.

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